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The research concerns H infinity control and focuses on substantially different parts of the subject, namely nonlinear systems, optimization theory and algorithms for frequency domain design and computer algebra tailored to systems and control research. For nonlinear plants, Helton-James made considerable progress on formulas for parameterizing all controllers. Also, for the very difficult measurement feedback problem they found a large class of "singular controllers" which can actually be implemented. We established that they have excellent stable equilibria. Work on optimization integrated raw H infinity methods with semidefinite programming algorithms. We expanded our computer algebra methods for reducing complicated sets of equations to nice sets of equations.

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H^∞ CONTROL FOR NONLINEAR AND LINEAR SYSTEMS AFOSR 49620-94-1-0185

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FINAL REPORT

Most of my work concerns H^∞ control but focuses on substantially different parts of the subject, namely, nonlinear systems, optimization theory and algorithms for frequency domain design, and computer algebra tailored to systems and control research.

Nonlinear systems

The modern approach to worst case design in the frequency domain arose from studies of amplifier design the "dual" problem of making a circuit dissipative using feedback. For linear systems key cases of this were solved in 1965 (SISO) by Youla and Saito and (MIMO) in 1976 by Helton. In the early 80's Zames and Francis formulated H^∞ control and solved the math problem by drawing on the earlier solutions to this circuits problem. In the beginning the subject of H^∞ control evolved quickly in significant part because key math problems were already reasonably understood by operator theorists. I participated in this earlier work (e.g. solved the MIMO H^∞ control problem with Zames and Francis, also Pearson and Chang) but at the same time begin pushing in new directions: nonlinear plants and an H^∞ approach to classical control.

Of the various solutions to CTRL one which is easy to implement and numerically sound is the Doyle-Glover-Kargonekar-Francis DGKF two Riccati equation solution. Consequently extending this to nonlinear plants is of considerable importance. Just prior to the contract period there has been considerable progress by Isidori and coworkers and by our group (Ball Helton Walker Zhan). Isidori et al find local sufficient conditions and compute (with Krener's software) power series solutions to model problems. All of these approaches assume something like the dimension of the compensator's state-space equals that of the controller state-space.

Evaluating performance of piecewise linear systems is an area where we made progress. We took a typical architecture (a la Campo-Morari) for a system with saturation and extracted one of the key computational difficulties. These systems are piecewise linear and continuous. Work with Ball showed that a key object for a dissipative system, called a storage function, must be continuous. We then made a natural compromise. The continuity of the storage function forces constraints which make analyzing such systems not a Linear Matrix Inequality. We found a sequence of steps which extracted the non LMI part and allowed one to solve the problem of determining performance of such systems by doing first an LMI check, then a side test then an LMI, etc.

General area of nonlinear H^∞ control In the general area of nonlinear H^∞ control we settled some basic theoretical issues. James and Baras have necessary and sufficient conditions on the H^∞ control problem. James and I have extended the basics of this. Under a saddle point assumption these reduce to cases also studied by van der Schaft and Basar. Krener has results of a similar tone. Vityaev and I gave the first theory on what PDE's arise when this saddle point structure fails.

This theory converts the problem of doing H^∞ control for a nonlinear system to solving two particular PDEs. One PDE which computes optimal feedback can be solved off line. One which gives the dynamics of the controller must be solved on line. Unfortunately, these are PDEs on the state space of the original plant (often a high dimensional space) so numerical solution faces what is called the curse of dimensionality.

Beating the online curse of dimensionality My work with James now indicates that, for the mixed sensitivity problem, the controller dynamics in practice might not suffer prohibitively from the curse of dimensionality. In this case the biggest parts of the computation can be done off line.

The main observations leading to this optimism (for the mixed sensitivity problem) are:

1. The controller PDE has some highly singular solutions p_t which equal $-\infty$ off of a small manifold M_t. This reduces the computational burden to handling functions on M_t.

2. For linear plants, the DGKF solutions can be put in various coordinates while for nonlinear plants many fewer coordinate changes are possible. If one uses the coordinates which are natural to the nonlinear theory in a linear H^∞ situation, then one gets exactly the singular solutions in (1). Indeed (1) gives the "central controller" which solves the linear H^∞ problem.

3. There is a solid theory in the nonlinear case when one uses smooth rather than singular solutions to the controller PDE.

4. What is needed is to extend this smooth function theory to singular functions. One thing we do know is that smooth solutions asymptotically converge to a singular solution p_∞, supported on the antistable manifold M_{antis} of the plant. This shows that if a smooth solution to the control problem exists, then p_∞ exists and has some good properties. Now p_∞ is the most natural initialization for the controller PDE and it produces our singular controller in (1). Hopefully, this controller solves the H^∞ problem, but this is far from being proved without very strong assumptions.

In conclusion, once a solution to the state feedback control problem for a plant whose antistable dimension is 0, 1, or maybe 2, there are now in principle formulas one could try for solving the measurement feedback part of the control problem. (The state feedback PDE remains oppressive.) Previously there was a reasonable theory (as in item 3) but no formulas. Now there are formulas but not much theory.

Current work with James on theory gives (probably too conservative) conditions along the lines required in (4) above on when the singular controllers solve the control problem.

Parameterization of all H^∞ controllers Parameterization of all linear H^∞ controllers for a system is equivalent to J inner/outer factorization of the system. The control and factorization problem for stable nonlinear systems was reduced to a Hamiltion-Jacobi-Bellman-Issacs (HJBI) equation by Ball and Helton (published in 1992). This left the unstable case open.

Helton and James

1. gave formal equations along [JB] lines for J inner/outer factors and prove properties which make their formulas look very promising,

2. tune the [JB] solution a bit to correct for an oversight and

3. tighten the necessary [JB] conditions to come much closer to a necessary and sufficient theory.

Optimization over H^∞

Much of my effort goes to studying a basic question of worst case frequency domain design where stability of the system is the key constraint. This is the H^∞ optimization problem which is crucial in several branches of engineering.

The fundamental H^∞ problem of control. First we state the core mathematics problem graphically. At each frequency ω we are given a set $\mathcal{S}_\omega(c) \subset \mathbf{C}^N$, called the *specification set.* The objective is to find a function T with no poles in the R.H.P. so that each $T(j\omega)$ belongs to $\mathcal{S}_\omega(c)$. In fact there is a simple picture to think of in connection with a design

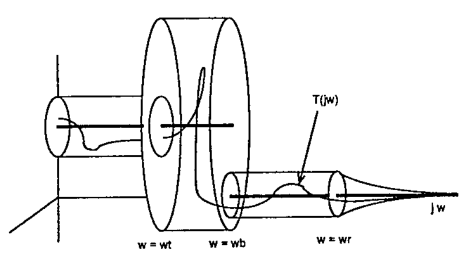

Figure 1

Typically there is a nested family of target sets $\mathcal{S}_\omega(c)$ parameterized by a performance level c. (The smaller the sets the better the performance.) For the *optimal* c a solution T exists but no solution exists for tighter specs.

The Horowitz templates of control can be transformed into this type of picture. When each $\mathcal{S}_\omega(c)$ is a "disk" this problem is solved by transformations of "classical pure" mathematics done in the late 1970's by Helton. Many different solutions to this problem in many different coordinates were worked out by engineers in the last 15 years since it is the subject of H^∞ control. Competing constraints and plant uncertainty lead immediately to spec sets which are not disks.

The graphical problem of Figure 1 can be formulated analytically in terms of a performance function Γ as

- (OPT) Given a positive valued function Γ on $\mathbf{R} \times \mathbf{C}^N$ (which is a performance measure), find $\gamma^* \geq 0$ and f^* in A_N which solve

$$\gamma^* = \inf_{f \in A_N} \sup_{\omega} \Gamma(\omega, f(j\omega)) \,.$$

and this of course is what one puts in a computer. Collaborators and I have a very broad based attack on the problem which addresses most aspects of it.

From qualitative theory to numerical algorithms and diagnostics While little was known about this problem 10 years ago there has been a lot of progress, and now we have a substantial amount of theory. We shall not sketch all that is known about OPT but emphasize that one of the most practical results on an optimization problem is characterization of the optimum, since this is the basis for numerics. We have, in the last few years, managed to push from high level theory to very effective computer algorithms.

Time domain constraints Merino, Walker and I were able to add time domain constraints to OPT and obtain optimality conditions extending those we already had for the OPT problem. Our result is easier to state on the unit disk Δ and the unit circle $\mathbf{T}$ rather than on the R.H.P. and the $j\omega$-axis. Also we state it only for the N=2 MIMO case.

We consider a constrained optimization problem, named **Constr-OPT**, where the minimization is done over analytic functions (f_1, f_2) that satisfy a given set of constraints:

$$\int_0^{2\pi} f_1 \overline{G_{1,\ell}} d\theta + \int_0^{2\pi} f_2 \overline{G_{2,\ell}} d\theta \geq 0, \qquad \ell = 1, \ldots, n$$

where the functions $G_{i,j}$ are analytic. Roughly the optimality condition for solutions to Constr-OPT is

Result 1 *Given Γ a smooth function and the constraints above and a smooth function T^* in H^∞ satisfying $a(e^{i\theta}) = \frac{\partial \Gamma}{\partial z}(e^{i\theta}, T^*(e^{i\theta}))$ is never 0 on $\mathbf{T}$. Necessary and sufficient conditions for T^* to be a local solution to* Constr-OPT *are*

I $\Gamma(e^{i\theta}, T^*(e^{i\theta}))$ is constant in $e^{i\theta}$.

II There exist F_1 and F_2 analytic on the disk, λ a positive function on the circle, and nonnegative constants $\kappa_1, \ldots, \kappa_n$ such that for all $e^{i\theta} \in \mathbf{T}$,

$$\frac{\partial \Gamma}{\partial z_1}(e^{i\theta}, T^*(e^{i\theta})) = \lambda(e^{i\theta}) \left(e^{i\theta} F_1(e^{i\theta}) + \kappa_1 \overline{G_{1,1}} + \ldots + \kappa_n \overline{G_{1,n}} \right)$$

$$\frac{\partial \Gamma}{\partial z_2}(e^{i\theta}, T^*(e^{i\theta})) = \lambda(e^{i\theta}) \left(e^{i\theta} F_2(e^{i\theta}) + \kappa_1 \overline{G_{2,1}} + \ldots + \kappa_n \overline{G_{2,n}} \right)$$

III A postivity condition on second derivatives of Γ.

When there are no time domain constraints this gives $\kappa_j = 0$ which is our workhorse result on OPT. Our analysis shows that Result 1 meshes well the basic OPT result for the purpose of constructing computer algorithms. We have worked out such algorithms and have begun testing.

Of independent interest is that all of this represents a new connection between engineering and an existing branch of the mathematics area Several Complex Variables.

Multiple performance objectives In H^∞ control one typically optimizes a supremum norm type of performance function. It has been known for many years that at optimum this performance function is frequency independent (i.e. flat).

We now consider two competing performances Γ_1 and Γ_2 which produces the 2-OPT problem.

Definition *A function $f^* \in H_N^\infty$ is called a* Pareto optimum *for Γ_1, Γ_2 if for each $f \in H_N^\infty$, we have*

$$\sup_\omega \Gamma_1(\omega, f) \geq \sup_\omega \Gamma_1(\omega, f^*) \quad \text{or} \quad \sup_\omega \Gamma_2(\omega, f) \geq \sup_\omega \Gamma_2(\omega, f^*) \,.$$

In other words, we cannot improve one of the competing performances without degrading others.

Andrei Vityaev and I showed that if the performance functions Γ_1, Γ_2 satisfy certain strong assumptions and if there are N designable subsystems $(f_1, ..., f_N)$ and l performance measures with $l \leq N$, then at a nondegenerate Pareto optimum $(f_1^*, ..., f_N^*)$ every performance is flat:

$$\Gamma_1(e^{i\theta}, f^*(\omega)) = const, \ldots, \Gamma_l(e^{i\theta}, f^*(\omega)) = const \,.$$

Besides flatness there are other "gradient alignment" conditions which must hold at an optimum. Thus we have the precise "first derivative" test for the most natural class of H^∞ Pareto optima.

MIMO performance measures

Most recent work with Merino and Walker has been in extending our study of OPT to the situation where the performance Γ is of the form

$$\Gamma(e^{i\theta}, f(e^{i\theta})) = \|\mathbf{\Gamma}(e^{i\theta}, f(e^{i\theta}))\|_{n\times n}$$

where $\mathbf{\Gamma}(e^{i\theta}, f)$ is a a smooth self-adjoint $n \times n$ matrix valued function. This representation is general enough to cover practically all situations that arise in applications, while at the same time allowing the non-smoothness of OPT to be treated as a feature of the matrix norm only. This leads to very effective analysis of the problem and to algorithms for solving it. A key result we obtained is the characterization of local solutions to OPT in this case:

Result 2 *Suppose that $\mathbf{\Gamma}$ is matrix valued. Under (mild) hypotheses, if f_* is a local solution to OPT for the performance $\|\mathbf{\Gamma}\|_{n\times n}$,*
then there exists a self adjoint $n \times n$ matrix valued function $\Psi = \Psi(e^{i\theta})$ such that

$$\begin{aligned}
(\gamma_* I - \mathbf{\Gamma}(\cdot, f_*)\,)\Psi_* &= 0 \\
P_{H_0^{2\perp}}[\ \mathrm{tr}\ (\ \tfrac{\partial \mathbf{\Gamma}}{\partial z_\ell}(\cdot, f_*)\Psi_*)] &= 0, \quad 1 \le \ell \le N \\
\int_0^{2\pi}\ \mathrm{tr}(\ \Psi_*\ )\tfrac{d\theta}{2\pi} &= 1 \\
\Psi_* &\ge 0 \\
\gamma_* I - \mathbf{\Gamma}(\cdot, f_*) &\ge 0
\end{aligned}$$

Our characterization of local solutions leads immediately to algorithms and optimality diagnostics. The algorithms, cases of which we have been working on for about five years, are of primal-dual semidefinite programming (SDP) type, so we are merging our H^∞ optimization theory with SDP in R^n.

We have found that in some cases it is possible to have second-order convergence rate, despite the fact that solutions are not unique. Numerical analysis of the algorithms is in progress.

Also, we are exploring with G. Balas the efficacy of placing diagnostics derived from our theory in the Matlab package μ-tools. Such diagnostics tell a practitioner how good an approximate answer to a μ-synthesis problem is, and aids further code development.

Computer algebra for systems research

If one reads a typical article on A,B,C,D systems in the control transactions one finds that most of the algebra involved is noncommutative. Thus for symbolic computing to have much impact on linear systems research one needs a program which will perform noncommuting operations. Mathematica, Macsyma and Maple (the 3 M's) do not. We have a package, NCAlgebra, which runs under Mathematica and does the basic operations, block matrix manipulations and other things. The package might be seen as a competitor to a yellow pad. Like Mathematica the emphasis is on interaction with the program and flexibility.

Mins and maxes of Hamiltonians Originally we wrote the package to do linear H^∞ control research. In particular, the main object in studying CTRL is an energy balance (game theoretic) Hamiltonian. One must compute critical points (maxes or mins) of this in W, a, b, c in various orders which is a routine but tedious process. Also any variation on the problem produces a new Hamiltonian and requires another tedious computation. NCAlgebra automates this. For example, if our Hamiltonian, labeled Ham, is quadratic in W and c, then

$$\begin{array}{ll}
critW = Crit[Ham, W]; & HnoW = Ham/.critW; \\
critWc = Crit[HnoW, c]; & HnoWc = HnoW/.critWc;
\end{array}$$

finds the critical point of Ham in W then in c and and evaluates Ham at these critical points.

Our research focuses on what types of "intelligence" to put in the package.

Simplification of messy formulas We are beginning to add serious automatic simplification commands to NCAlgebra. Stankus, Wavrik, and I are now doing research in computer simplification for A, B, C, D type

linear systems, in a highly noncommutative setting. The objective is in each particular situation to find a list of simplifying rules. A complete list of rules (called a Gröbner basis GB) has the property that if it is applied to an expression until nothing changes then the expression is as simple as possible in a certain sense. Recently, Wavrik and I obtained

> *For the formulas which occur in studying energy conserving (lossless) systems. the GB while infinite can be summarized as a list of 32 rules some of which depend on an integer parameter. It is a powerful tool for studying a particular class of systems. The list was discovered last year and actually proved (with Stankus) to be a GB very recently.*

A subset of these rules is now in a function NCSimplifyRational[expression] inside our NCAlgebra package. They are very effective on a limited class of expressions but even that makes them very useful.

Discovering formulas Current work developes something, we call a strategy; it is in a primitive stage. These are methods for "discovering" algebraic theorems and formulas semiautomatically.

This, like the simplification methods, is based on what is called a noncommutative Gröbner Basis Algorithm (GBA). The GBA has the effect of systematically eliminating unknowns so as to put a system of polynomial equations into "triangular form". The commutative case of the GBA is the core of the "Solve" commands of the 3 M's and it is used in many fields.

The input to a GBA is (1) a list of knowns, (2) a list of unknowns (together with priorities for eliminating them) and (3) a collection of equations in these knowns and unknowns. A *strategy* is : run the GBA, sort the output into equations involving only one unknown (say one contains only x_1), the user must now make a decision about equations in x_1 (e.g., this is a Riccati so I shall not try to simplify it, but leave it for Matlab). Now the user declares the unknown x_1 to be known and runs the GBA again. Sometimes one needs a *2-strategy* in that the key is equations in 2 unknowns. The point is to isolate and to minimize what the user must do. We organize strategies via a "spreadsheet" for discovering theorems.

We are under the impression that many theorems in engineering systems theory are of this type. At the beginning of "discovering" a theorem, a problem is presented as a large system of matrix equations. Often when viewing the output of the GB algorithm, one can see what additional hypotheses should be added to produce a useful theorem and what the relevant matrix quantities are. Our efforts are in a primitive stage and the brevity of this exposition suppresses some of the advantages and some of the difficulties.

Example Suppose we are given a collection of equations involving matricies. For example,

$$(FAC) \qquad \begin{gathered} Am_1 - m_1 a - m_2 f c = 0 \\ Am_2 - m_2 e = 0 \\ B - m_1 b - m_2 f = 0 \\ -c + C\, m_1 = 0 \\ -g + C\, m_2 = 0 \\ im_1\, m_1 - 1 = 0 \\ im_1\, m_2 = 0 \\ im_2\, m_1 = 0 \\ im_2\, m_2 - 1 = 0 \\ m_1\, im_1 + m_2\, im_2 - 1 = 0 \end{gathered}$$

where A, B and C are known and the lower case letters a, b, c, e, f, g, m_1, m_2, im_1 and im_2 are unknown. We want to solve (FAC) for the unknowns We use this as an illustration because it corresponds to the well known problem of factoring a system $[A, B, C, 1]$ "minimally" into the product of two systems $(a, b, c, 1)$ and $(e, f, g, 1)$. The unknown matrix $(m_1\ m_2)$ corresponds to an isomorphism between the statespace of the (unknown) product system and the statespace of the (known) original system. A well known theorem says factoring is possible iff there exist complementary projections P_1, P_2 satisfying

$$P_1(A - BC)P_1 = (A - BC)P_1 \quad \text{and} \quad P_2 A P_2 = A P_2 . \tag{1}$$

We now apply a strategy to see how one might discover this theorem. The input is the equations (FAC), together with declaration of A, B, C as knowns and the remaining variables as unknowns. Here is the spreadsheet which the computer generates:

SPREADSHEET

```
=============================================
 THE ALGORITHM HAS SOLVED FOR: {c,g,a,b,e,f}
=============================================

The corresponding rules are the following:
a->im1**A**m1       b->im1**B     c->C**m1
e->im2**A**m2       f->im2**B     g->C**m2

=============================================
======= UNDIGESTED RELATIONS APPEAR BELOW ===
=============================================

THE FOLLOWING VARIABLES HAVE NOT BEEN SOLVED
FOR: {im1,im2,m1,m2}

The expressions with unknown variables
{im1,m1} and knowns {A,B,C}
---------------------------------------
im1**m1->1

m1**im1**B**C**m1->  -A**m1 +         <===
   B**C**m1 + m1**im1**A**m1

m1**im1**B**C**B**C**m1->A^2**m1 - A**B**C**m1 -
   B**C**A**m1 + B**C**B**C**m1 - m1**im1**A^2**m1 +
   m1**im1**A**B**C**m1 + m1**im1**B**C**A**m1

The expressions with unknown variables {im1,m2}
and knowns {A}
---------------------------------------
im1**m2->0
im1**A**m2->0

The expressions with unknown variables {im2,m1}
and knowns {A,B,C}
---------------------------------------
im2**m1->0
im2**B**C**m1->im2**A**m1

The expressions with unknown variables {im2,m2}
and knowns {A}
---------------------------------------
im2**m2->1
m2**im2**A**m2->A**m2             <===

The expressions with unknown variables
{im1,im2,m1,m2} and knowns {B}
---------------------------------------
m2**im2**B-> B - m1**im1**B
m2**im2->1 - m1**im1
```

The unknowns a,b,c,e,f and g are solved for. There are no equations in 1 unknown. There are 4 categories of equations in 2 unknowns. A user must observe that the equations which I marked with <=== each transform to equations in one variable P_1 (respectively P_2) when one makes the assignments:

$$P_1 = m_1\, im_1 \quad \text{and} \quad P_2 = m_2\, im_2\,. \tag{2}$$

Run GBA again with (2) added and P_1, P_2 declared known. The resulting spreadsheet is much like the one above but has the added piece

```
          The expressions with unknown variables { } and knowns {A,B,C,P1,P2}
              ---------------------------------------
P2->1 + -P1   P1**A**P1->P1**A     P1**B**C**P1-> -A**P1+P1**A+B**C**P1     P1^2->P1

P1**A^2**P1->P1**A^2                                            <=== REDUNDANT
P1**B**C**B**C**P1->A^2**P1 - P1**A^2 - A**B**C**P1 - B**C**A**P1 +
   B**C**B**C**P1 + P1**A**B**C**P1 + P1**B**C**A**P1              <=== REDUNDANT
```

You see the first line is the condition (1) of the classical theorem (plus stating that P_1, P_2 are complementary projections) which immediately proves one side of the theorem. It takes one more GBA run to remove the redundant equations and thereby prove the converse direction.

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[HJ96] J. W. Helton, M. R. James. J-Inner/Outer Factorization for Bilinear Systems, CDC96, p. 3788.

[HJ] J.W. Helton and M. R. James, "On the Stability of the Information State System," Sys Control Letters vol 29 (1996) p61-72

PERSONNEL SUPPORTED

Faculty	**5**
Helton, J.W.	PI
Gu, Caixing	UC Irvine
Dym, H.	Weizmann Institute
Merino, O	University of Rhode Island
Young, Nicholas	Lancaster University, England

Post Doctorates 2

Stankus, M
James, Matthew

Graduate Students 5

Iakoubovski, Mikhail
Myers, Julia
Slobodan, Kojcinovic
Vityaev, Andrei
Vikram , Srimurthy

Undergraduates/Other 8

Files, James
Herrero, Pablo
Mager, Michael
Moore, Michael
Rowell, Eric
Schneider, Kurt
Shih, Victor
Yoshinobu, Stan

ADVISORY FUNCTIONS

NOSC (San Diego)
They are looking into H infinityÉ identification of antenna response functions. I advise on the effort.

General Atomic Corp (Controled Fusion group) -- I give lectures and advice on control systems for Tokamaks.

TRANSITIONS

We have two computer programs which run under Mathematica which are publicly available.

NCAlgebra, our non commuting algebra package, has potential applications in many fields (request from ncalg@osiris.ucsd.edu). We are the main providers of Mathematica noncommutative capability. They appear to be recommending it widely.

OPTDesign our classical control program is available from us (send request to anopt@osiris.ucsd.edu). We do not intend to start pushing it heavily until our book is published, since this is the only account which ties everything together.

Another level of transfer is from pure to applied mathematics. For example, in the last decade progress in H infinityÉ control was expedited by close connections with operator theorists who were originally in pure mathematics but who now work on the mathematics of engineering systems. This originated with discoveries by DeWilde, Fuhrmann and I which were made in the early 1970's.

The work on optimization over analytic functions represents a new connection between engineering and an existing branch of several complex variables. Now little collaboration exists between workers in these areas. A bi-product of our development of (OPT) is possibly that a new group of pure mathematicians will become interested in engineering.

Also, we are exploring with G. Balas the efficacy of placing diagnostics derived from our theory in the Matlab package mu-tools. This tells users how good an approximate answer to a mu-synthesis problem is. Also it aids further code development.

Ford Motor Co. contributed $10,000 to our research.

INTERACTIONS

A. Participation, Presentations at Professional Meetings, Conferences, Seminars, etc..

AMS Regional Meeting, Virginia, (Special session speaker)	11/94
CDC '94, Florida (presented papers with the following) James Merino Stankus	12/94
Air Force Contractors Meeting, MN	6/95
Engineering Conference at Stanford University, Palo Alto, CA (T. Kailath organizer)	6/95
Nonlinear Control Conf (NOLCOS) Tahoe City. CA (presented papers the following) Vityaev James (presented plenary talk on joint work)	6/95
Australia National Univ. ("month" long visitor,talk, July 9-30)	7/95
MSRI (3 month visitor)	Fall 1995
CDC '95 New Orleans (presented papers with the following) James Merino Stankus Vityaev	1995
Lecture at Information Systems Lab, Stanford University	1/96
Talk at Smart Materials Conference (SPIE), San Diego.	2/96
Colloquium at the Mathematics Department, Cal Tech.	5/96
Seminar at the center for Dynamical Systems, Cal Tech.	5/96
Talk -Workshop on Operator Theory, Indianna Univ.	6/96
Thu 20 Semipleniary talk, MTNS (Mathematica Theory of Networks andSystems) Two session talks	6/96
CDC'96 - Kobe, Japan (presented papers with the following) James Yuliar Merino Stankus	12/96
Lectures in Kokotovic Seminar at UC Santa Barbara	2/97

NEW DISCOVERIES --- Patents and Inventions

None

HONORS/AWARDS (Lifetime)

Professional Distinctions - Plenary addresses:

Mathematical Theory of Networks and Systems	1979
AMS Annual Meeting	1980
European Conference on Circuit Theory and Design	1981
Mathematical Theory of Networks and Systems	1983
Toeplitz Lecture, University of Tel Aviv	1985
Principal Lecturer, CBMS Regional Conference - NE	1985
Coble Lectures, University of Illinois	1986
SIAM Conference LASSC (Systems of Applications of Matrices)	1986
NSF panel to review the state of classical complex analysis,	
Co-organizer: Signal Processing IMA 10-week workshop	1988
Mathematical Theory of Networks and Systems	1989
Lake Como Lectures (CIME)	1990
Great Plains Operator Theory Symposium	1992
Lecturers on Nonlinear Control - Taiwan	1992
Mathematical Theory of Networks and Systems	1993
Mathematical Theory of Networks and Systems (semi-pleniary)	1996

Professional Distinctions - Other

Guggenheim Fellow	1985
Outstanding paper, IEEE Control Society	1986

Associate Editor, *Journal of Operator Theory*
Associate Editor, *Journal of Operator Theory and Integral Equations*
Associate Editor, *Journal Mathematical Analysis and Applications*
Associate Editor, *Nonlinear and Robust Control*
Associate Editor, *CRC book series*
Associate Editor, *Fourier Analysis and Applications*